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Quantum transport through double-dot Aharonov-Bohm interferometry in Coulomb blockade regime

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Abstract. Transport through two quantum dots laterally embedded in Aharonov-Bohm interferometry with infinite intradot and arbitrary interdot Coulomb repulsion is analyzed in the weak coupling and Coulomb blockade regime. By employing the modified quantum rate equations and the slave-boson approach, we establish a general dc current formula at temperatures higher than the Kondo temperature for the case that the spin degenerate levels of two dots are close to each other. For further discussion, we examine two simple examples for identical dots - no doubly occupied states and no empty state. In the former, completely destructive coherent transport and phase locking appear at magnetic flux $\Phi = \Phi_0/2$ and $\Phi = 0$ respectively; in the latter, partially coherent transport exhibits an oscillation with magnetic flux having a period of Φ_0.

PACS. 73.21.La Quantum dots – 73.23.-b Electronic transport in mesoscopic systems – 73.23.Hk Coulomb blockade and single-electron tunneling.

1 Introduction

Quantum dot (QD), a tiny engineered device accommodating a single electron or a few ones in three-dimensionally confined space, acts not only as a crucial ingredient for the realization of solid state quantum computation but also as a convenient tool to explore the effect of strong correlation manifested by discrete energy levels. Very rich phenomena, such as resonant tunneling, Coulomb blockade and the Kondo effect, arise in different circumstances to intrigue experimentalists and theorists. To address the phase coherence of the transport, the QD or QDs are embedded in various Aharonov-Bohm (AB) geometries. Till now single dot [1–5] or double dots [6–8] in two-terminal AB interferometer have been realized in experiments and the current oscillation of magnetic flux has been observed.

In this paper, we consider two quantum dots parallely connected to two metallic reservoirs with magnetic flux penetrating the enclosed area. Due to the complexity of this system, most of the previous research concentrated on the noninteracting case to give the exact and general results of transport and to find effects of magnetic flux, position and difference of QDs energy levels, and band widths of coupling strength [9–15]. There were only a few theoretical studies of the interacting systems, which focused either on intradot correlation with spin for two identical dots [16,12,17], or on interdot correlation without spin for two different dots [19,18], or on the case having large intradot and interdot correlation without spin for two identical dots [12]. Reference [20] is an exception where the intradot and interdot Coulomb repulsion, spin configuration and disparity of two dot levels were discussed at zero temperature covering the strong coupling Kondo regime. The present paper also deals with intradot and interdot Coulomb correlation, spin configuration and level disparity but is concerned with the Coulomb blockade regime at temperatures higher than the Kondo temperature.

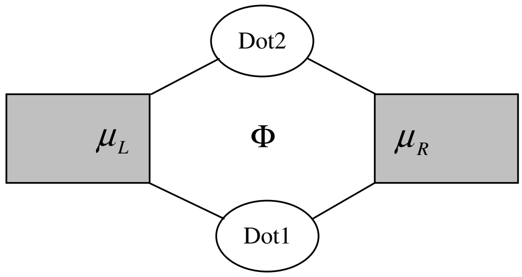

Fig. 1. Schematic of a double-dot system in a parallel configuration between two leads in the presence of Aharonov-Bohm magnetic flux.

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For the description of quantum transport through double-dot system, the "classical" rate equations must be modified for nondiagonal density matrix elements responsible for transitions between isolated quantum states [21–25]. Recently a new version of the modified quantum rate equations have been derived [26] to study the transport of an interacting system utilizing the slave-boson technique introduced by Zou and Anderson [27] and incorporating the nonequilibrium Green's functions. The solutions are equivalent to the lowest-order gradient expansion, which is a good approximation for sequential resonant tunneling [28], and essentially accordant with previous analyses. Using this kind of quantum rate equations, one is able to discuss the Coulomb correlation effect at arbitrary temperature. In the present paper, we apply this method to analyze a double-dot AB interferometer with interdot and intradot Coulomb repulsion, considering the lowest-order of the dot-lead coupling strength in transport.

2 Formulation

The Hamiltonian of two tunneling coupled quantum dots parallely connected to left and right leads with the presence of magnetic flux is described by the genetic Anderson model:

$$\begin{aligned}H = &\sum_{\alpha k\sigma} \epsilon_{\alpha k\sigma} c^\dagger_{\alpha k\sigma} c_{\alpha k\sigma} + \epsilon_1 \sum_\sigma c^\dagger_{1\sigma} c_{1\sigma} + U n_{1\uparrow} n_{1\downarrow} \\ &+\epsilon_2 \sum_\sigma c^\dagger_{2\sigma} c_{2\sigma} + U n_{2\uparrow} n_{2\downarrow} + U' \sum_{\sigma\sigma'} n_{1\sigma} n_{2\sigma'} \\ &+ \sum_{k\sigma} (t_{L1\sigma} c^\dagger_{Lk\sigma} c_{1\sigma} + t_{R1\sigma} c^\dagger_{Rk\sigma} c_{1\sigma} + \text{h.c.}) \\ &+ \sum_{k\sigma} (t_{L2\sigma} c^\dagger_{Lk\sigma} c_{2\sigma} + t_{R2\sigma} c^\dagger_{Rk\sigma} c_{2\sigma} + \text{h.c.}), \qquad (1)\end{aligned}$$

where $c^\dagger_{1(2)\sigma}$ ($c_{1(2)\sigma}$) and $c^\dagger_{\alpha k\sigma}$ ($c_{\alpha k\sigma}$) are creation (annihilation) operators of electrons in the dots 1(2) and in the left and right leads ($\alpha = L, R$) with spin σ. Each dot has a single spin degenerate orbital level $\epsilon_{1(2)}$ and an infinite on-site Coulomb repulsion U and simultaneously there is an arbitrary finite interdot electrostatic correlation U' between them. We only consider the two dot levels are very close to each other, $\epsilon_{2(1)} = \epsilon_d \pm \epsilon/2$, with a small variation ϵ. The effect of AB flux Φ is taken into account in the tunneling amplitude $t_{\alpha 1(2)\sigma}$ by $\varphi = 2\pi\Phi/\Phi_0$ with the flux quantum $\Phi_0 = h/e$. In the Peierls gauge, one generally chooses $t^*_{L1\sigma} = t_{L2\sigma} = t^*_{R2\sigma} = t_{R1\sigma} = |t|e^{i\varphi/4}$ [10,12]. The αth lead is supposed to be Fermi liquids in equilibrium state and has the same coupling strength function with two dots $\Gamma_{\alpha\sigma}(\omega) = 2\pi \sum_{k\alpha} |t_{\alpha 1(2)\sigma}|^2 \delta(\omega - \epsilon_{\alpha k\sigma})$.

According to the slave-particle approach originated by Zou and Anderson, we introduce auxiliary operators $e^\dagger$, $f^\dagger_{1\sigma}$ ($f^\dagger_{2\sigma}$), $d^\dagger_{\sigma\sigma'}$ to stand for the possible states of two dots as a whole: empty state $|0\rangle_1|0\rangle_2$, singly occupied state $|\sigma\rangle_1|0\rangle_2$ ($|0\rangle_1|\sigma\rangle_2$), and doubly occupied state $|\sigma\rangle_1|\sigma'\rangle_2$ respectively. In the slave-boson representation, the electron operators of each dot are substituted by the slave-boson operators $e^\dagger$, $d^\dagger$ and the pseudo-fermion operators $f^\dagger_{1\sigma}$, $f^\dagger_{2\sigma}$ [27,26]:

$$c_{1\sigma} = e^\dagger f_{1\sigma} + \sum_{\sigma'} f^\dagger_{2\sigma'} d_{\sigma\sigma'}, \; c_{2\sigma} = e^\dagger f_{2\sigma} + \sum_{\sigma'} f^\dagger_{1\sigma'} d_{\sigma'\sigma}, \qquad (2)$$

with the completeness constraint $e^\dagger e + \sum_\sigma (f^\dagger_{1\sigma} f_{1\sigma} + f^\dagger_{2\sigma} f_{2\sigma}) + \sum_{\sigma\sigma'} d^\dagger_{\sigma\sigma'} d_{\sigma\sigma'} = 1$, for which these operators must be correctly quantized to make the sum rule for the physical electron valid and the commutators between them are satisfying [29,26]:

$$\begin{aligned}&ee^\dagger = 1, \; d_{\sigma_1\sigma_2} d^\dagger_{\sigma'_1\sigma'_2} = \delta_{\sigma_1\sigma'_1}\delta_{\sigma_2\sigma'_2}, \; f_{i\sigma} f^\dagger_{j\sigma'} = \delta_{ij}\delta_{\sigma\sigma'}, \\ &ed^\dagger_{\sigma\sigma'} = ef^\dagger_{i\sigma} = f_{i\sigma} e^\dagger = f_{i\sigma} d^\dagger_{\sigma'\sigma''} = d_{\sigma\sigma'} e^\dagger = d_{\sigma'\sigma''} f^\dagger_{i\sigma} = 0. \qquad (3)\end{aligned}$$

So the effective Hamiltonian is written in terms of these instrumental state operators:

$$\begin{aligned}H_{eff} = &\sum_{\alpha k\sigma} \epsilon_{\alpha k\sigma} c^\dagger_{\alpha k\sigma} c_{\alpha k\sigma} + \epsilon_1 \sum_\sigma f^\dagger_{1\sigma} f_{1\sigma} + \epsilon_2 \sum_\sigma f^\dagger_{2\sigma} f_{2\sigma} \\ &+(\epsilon_1 + \epsilon_2 + U') \sum_{\sigma\sigma'} d^\dagger_{\sigma\sigma'} d_{\sigma\sigma'} \\ &+ \sum_{k\sigma} [t_{L1\sigma} c^\dagger_{Lk\sigma} (e^\dagger f_{1\sigma} + \sum_{\sigma'} f^\dagger_{2\sigma'} d_{\sigma\sigma'}) \\ &+t_{R1\sigma} c^\dagger_{Rk\sigma} (e^\dagger f_{1\sigma} + \sum_{\sigma'} f^\dagger_{2\sigma'} d_{\sigma\sigma'}) + \text{h.c.}] \\ &+ \sum_{k\sigma} [t_{L2\sigma} c^\dagger_{Lk\sigma} (e^\dagger f_{2\sigma} + \sum_{\sigma'} f^\dagger_{1\sigma'} d_{\sigma'\sigma}) \\ &+t_{R2\sigma} c^\dagger_{Rk\sigma} (e^\dagger f_{2\sigma} + \sum_{\sigma'} f^\dagger_{1\sigma'} d_{\sigma'\sigma}) + \text{h.c.}], \qquad (4)\end{aligned}$$

and the elements of projection operator for the density matrix are expressed as $\hat\rho_{00} = |0\rangle_1|0\rangle_{22}\langle 0|_1\langle 0| = e^\dagger e$, $\hat\rho_{11\sigma} = |\sigma\rangle_1|0\rangle_{22}\langle 0|_1\langle\sigma| = f^\dagger_{1\sigma} f_{1\sigma}$, $\hat\rho_{22\sigma} = |0\rangle_1|\sigma\rangle_{22}\langle\sigma|_1\langle 0| = f^\dagger_{2\sigma} f_{2\sigma}$, $\hat\rho_{dd\sigma\sigma'} = |\sigma\rangle_1|\sigma'\rangle_{22}\langle\sigma'|_1\langle\sigma| = d^\dagger_{\sigma\sigma'} d_{\sigma\sigma'}$, and $\hat\rho_{12\sigma} = |0\rangle_1|\sigma\rangle_{22}\langle 0|_1\langle\sigma| = f^\dagger_{2\sigma} f_{1\sigma}$, $\hat\rho_{21\sigma} = |\sigma\rangle_1|0\rangle_{22}\langle\sigma|_1\langle 0| = f^\dagger_{1\sigma} f_{2\sigma}$.

Supposing the left and right leads are made from the identical material and the effective coupling strength is constant for the energy range of interest $\Gamma_{L,R\sigma}(\omega) = \Gamma_{L,R\sigma}$. The Heisenberg equations of motion for the six projection operator elements derived from the effective Hamiltonian form the basic equations. In the statistical expectations of these equations, after the Langreth analytic continuation to decouple the dots and the leads interaction terms, the rate equations are acquired in the wide band limit:

$$\begin{aligned}\dot\rho_{00} = &\frac{-i}{2\pi} \int d\omega \sum_\sigma [A_\sigma (G^>_{e11\sigma} + G^>_{e22\sigma}) + C_\sigma (G^<_{e11\sigma} + G^<_{e22\sigma}) \\ &+ B_\sigma G^>_{e12\sigma} + B^*_\sigma G^>_{e21\sigma} + D_\sigma G^<_{e12\sigma} + D^*_\sigma G^<_{e21\sigma}], \qquad (5)\end{aligned}$$

$$\dot{\rho}_{11\sigma} = \frac{i}{2\pi}\int d\omega(A_\sigma G^>_{e11\sigma} + 1/2B_\sigma G^>_{e12\sigma} + 1/2B^*_\sigma G^>_{e21\sigma} + C_\sigma G^<_{e11\sigma} + 1/2D_\sigma G^<_{e12\sigma} + 1/2D^*_\sigma G^<_{e21\sigma}) - \frac{i}{4\pi}\int d\omega \sum_{\sigma'\sigma''}[A_{\sigma'}(G^>_{d11\sigma'\sigma\sigma''} + G^>_{d11\sigma'\sigma''\sigma}) + B_{\sigma'}G^{''>}_{d21\sigma'\sigma''\sigma} + B^*_{\sigma'}G^>_{d12\sigma\sigma'\sigma''} + C_{\sigma'}(G^<_{d11\sigma'\sigma\sigma''} + G^<_{d11\sigma'\sigma''\sigma}) + D_{\sigma'}G^{''<}_{d21\sigma'\sigma''\sigma} + D^*_{\sigma'}G^<_{d12\sigma\sigma'\sigma''}], \quad (6)$$

$$\dot{\rho}_{22\sigma} = \frac{i}{2\pi}\int d\omega(A_\sigma G^>_{e22\sigma} + 1/2B_\sigma G^>_{e12\sigma} + 1/2B^*_\sigma G^>_{e21\sigma} + C_\sigma G^<_{e22\sigma} + 1/2D_\sigma G^<_{e12\sigma} + 1/2D^*_\sigma G^<_{e21\sigma}) - \frac{i}{4\pi}\int d\omega \sum_{\sigma'\sigma''}[A_{\sigma'}(G^>_{d22\sigma'\sigma\sigma''} + G^>_{d22\sigma'\sigma''\sigma}) + B_{\sigma'}G^{''>}_{d21\sigma'\sigma\sigma''} + B^*_{\sigma'}G^>_{d12\sigma''\sigma'\sigma} + C_{\sigma'}(G^<_{d22\sigma'\sigma\sigma''} + G^<_{d22\sigma'\sigma''\sigma}) + D_{\sigma'}G^{''<}_{d21\sigma'\sigma\sigma''} + D^*_{\sigma'}G^<_{d12\sigma''\sigma'\sigma}], \quad (7)$$

$$\dot{\rho}_{12\sigma} = \frac{i}{4\pi}\int d\omega[2A_\sigma G^>_{e12\sigma} + B^*_\sigma(G^>_{e11\sigma} + G^>_{e22\sigma}) + 2C_\sigma G^<_{e12\sigma} + D^*_\sigma(G^<_{e11\sigma} + G^<_{e22\sigma})] - \frac{i}{4\pi}\int d\omega \sum_{\sigma'\sigma''}[A_{\sigma'}(G^>_{d21\sigma'\sigma''\sigma} + G^{'>}_{d21\sigma\sigma'\sigma''}) + B^*_{\sigma'}(G^{'>}_{d11\sigma'\sigma''\sigma} + G^{'>}_{d22\sigma\sigma'\sigma''}) + C_{\sigma'}(G^<_{d21\sigma'\sigma''\sigma} + G^{'<}_{d21\sigma\sigma'\sigma''}) + D^*_{\sigma'}(G^{'<}_{d11\sigma'\sigma''\sigma} + G^{'<}_{d22\sigma\sigma'\sigma''})] + i(\epsilon_2 - \epsilon_1)\rho_{12\sigma}, \quad (8)$$

$$\dot{\rho}_{dd\sigma\sigma} = \frac{i}{4\pi}\int d\omega \sum_{\sigma'}[A_\sigma(G^>_{d11\sigma\sigma\sigma'} + G^>_{d11\sigma\sigma'\sigma} + G^>_{d22\sigma\sigma\sigma'} + G^>_{d22\sigma\sigma'\sigma}) + B_\sigma(G^{''>}_{d21\sigma\sigma\sigma'} + G^{''>}_{d21\sigma\sigma'\sigma}) + B^*_\sigma(G^>_{d12\sigma'\sigma\sigma} + G^>_{d12\sigma\sigma\sigma'}) + C_\sigma(G^<_{d11\sigma\sigma\sigma'} + G^<_{d11\sigma\sigma'\sigma} + G^<_{d22\sigma\sigma\sigma'} + G^<_{d22\sigma\sigma'\sigma}) + D_\sigma(G^{''<}_{d21\sigma\sigma\sigma'} + G^{''<}_{d21\sigma\sigma'\sigma}) + D^*_\sigma(G^<_{d12\sigma'\sigma\sigma} + G^<_{d12\sigma\sigma\sigma'})], \quad (9)$$

$$\dot{\rho}_{dd\sigma\bar{\sigma}} = \frac{i}{4\pi}\int d\omega \sum_{\sigma'}[A_{\bar{\sigma}}(G^>_{d11\bar{\sigma}\sigma\sigma'} + G^>_{d11\bar{\sigma}\sigma'\sigma}) + A_\sigma(G^>_{d22\sigma\bar{\sigma}\sigma'} + G^>_{d22\sigma\sigma'\bar{\sigma}}) + B_\sigma G^{''>}_{d21\sigma\bar{\sigma}\sigma'} + B_{\bar{\sigma}}G^{''>}_{d21\bar{\sigma}\sigma'\sigma} + B^*_\sigma G^>_{d12\sigma'\sigma\bar{\sigma}} + B^*_{\bar{\sigma}}G^>_{d12\sigma\bar{\sigma}\sigma'} + C_{\bar{\sigma}}(G^<_{d11\bar{\sigma}\sigma\sigma'} + G^<_{d11\bar{\sigma}\sigma'\sigma}) + C_\sigma(G^<_{d22\sigma\bar{\sigma}\sigma'} + G^<_{d22\sigma\sigma'\bar{\sigma}}) + D_\sigma G^{''<}_{d21\sigma\bar{\sigma}\sigma'} + D_{\bar{\sigma}}G^{''<}_{d21\bar{\sigma}\sigma'\sigma} + D^*_\sigma G^<_{d12\sigma'\sigma\bar{\sigma}} + D^*_{\bar{\sigma}}G^<_{d12\sigma\bar{\sigma}\sigma'}], \quad (10)$$

here $A_\sigma = f_L(\omega)\Gamma_{L\sigma} + f_R(\omega)\Gamma_{R\sigma}$, $C_\sigma = [1-f_L(\omega)]\Gamma_{L\sigma} + [1-f_R(\omega)]\Gamma_{R\sigma}$, $B_\sigma = f_L(\omega)\Gamma_{L\sigma}e^{-i\varphi/2} + f_R(\omega)\Gamma_{R\sigma}e^{i\varphi/2}$, $D_\sigma = [1-f_L(\omega)]\Gamma_{L\sigma}e^{-i\varphi/2} + [1-f_R(\omega)]\Gamma_{R\sigma}e^{i\varphi/2}$, with the Fermi distribution function $f_\alpha(\omega) = 1/(e^{\beta(\omega-\mu_\alpha)}+1)$, μ_α the chemical potential of the αth lead at temperature $1/\beta$.

For a symmetric system $\Gamma_{L\sigma} = \Gamma_{R\sigma} = \Gamma_\sigma$, the dc current of the stationary state can be symmetrized $I = (I_L - I_R)/2$ and $I_\alpha(\alpha = L, R)$ is obtained from the average of time derivative of electron number operator of the αth lead:

$$I = \frac{ie}{2h}\int d\omega \sum_\sigma \Gamma_\sigma\{[f_L(\omega) - f_R(\omega)][G^>_{e11\sigma} + G^>_{e22\sigma} - G^<_{e11\sigma} - G^<_{e22\sigma} + \sum_{\sigma'\sigma''}(G^>_{d11\sigma\sigma'\sigma''} + G^>_{d22\sigma\sigma'\sigma''} - G^<_{d11\sigma\sigma'\sigma''} - G^<_{d22\sigma\sigma'\sigma''})] + \cos\frac{\varphi}{2}[f_L(\omega) - f_R(\omega)] \times [G^>_{e12\sigma} + G^>_{e21\sigma} - G^<_{e12\sigma} - G^<_{e21\sigma} + \sum_{\sigma'\sigma''}(G^{''>}_{d21\sigma\sigma'\sigma''} + G^>_{d12\sigma''\sigma\sigma'} - G^{''<}_{d21\sigma\sigma'\sigma''} - G^<_{d12\sigma''\sigma\sigma'})] + i\sin\frac{\varphi}{2}[f_L(\omega) + f_R(\omega)][G^>_{e21\sigma} - G^>_{e12\sigma} + \sum_{\sigma'\sigma''}(G^>_{d12\sigma''\sigma\sigma'} - G^{''>}_{d21\sigma\sigma'\sigma''})] + 2i\sin\frac{\varphi}{2}\left[1 - \frac{f_L(\omega) + f_R(\omega)}{2}\right] \times [G^<_{e21\sigma} - G^<_{e12\sigma} + \sum_{\sigma'\sigma''}(G^<_{d12\sigma''\sigma\sigma'} - G^{''<}_{d21\sigma\sigma'\sigma''})]\}. \quad (11)$$

This expression is similar with that of Ref.[12] where the interdot correlation was not included. The Green's functions quoted in the above are defined as follows:

$$\begin{aligned}
G_{eii\sigma} &= \langle\langle e^\dagger(t)f_{i\sigma}(t)|f^\dagger_{i\sigma}(t')e(t')\rangle\rangle,\\
G_{eij\sigma} &= \langle\langle e^\dagger(t)f_{i\sigma}(t)|f^\dagger_{j\sigma}(t')e(t')\rangle\rangle,\\
G_{d11\sigma'\sigma\sigma''} &= \langle\langle f^\dagger_{1\sigma}(t)d_{\sigma\sigma'}(t)|d^\dagger_{\sigma''\sigma'}(t')f_{1\sigma''}(t')\rangle\rangle,\\
G'_{d11\sigma'\sigma''\sigma} &= \langle\langle f^\dagger_{1\sigma''}(t)d_{\sigma''\sigma'}(t)|d^\dagger_{\sigma'\sigma}(t')f_{1\sigma}(t')\rangle\rangle,\\
G_{d22\sigma'\sigma\sigma''} &= \langle\langle f^\dagger_{2\sigma}(t)d_{\sigma'\sigma}(t)|d^\dagger_{\sigma'\sigma''}(t')f_{2\sigma''}(t')\rangle\rangle,\\
G'_{d22\sigma\sigma'\sigma''} &= \langle\langle f^\dagger_{2\sigma}(t)d_{\sigma\sigma'}(t)|d^\dagger_{\sigma'\sigma''}(t')f_{2\sigma''}(t')\rangle\rangle,\\
G_{d21\sigma'\sigma''\sigma} &= \langle\langle f^\dagger_{2\sigma''}(t)d_{\sigma'\sigma''}(t)|d^\dagger_{\sigma'\sigma}(t')f_{1\sigma}(t')\rangle\rangle,\\
G'_{d21\sigma\sigma'\sigma''} &= \langle\langle f^\dagger_{2\sigma}(t)d_{\sigma\sigma'}(t)|d^\dagger_{\sigma''\sigma'}(t')f_{1\sigma''}(t')\rangle\rangle,\\
G''_{d21\sigma'\sigma\sigma''} &= \langle\langle f^\dagger_{2\sigma}(t)d_{\sigma'\sigma}(t)|d^\dagger_{\sigma''\sigma'}(t')f_{1\sigma''}(t')\rangle\rangle,\\
G_{d12\sigma\sigma'\sigma''} &= \langle\langle f^\dagger_{1\sigma}(t)d_{\sigma\sigma'}(t)|d^\dagger_{\sigma'\sigma''}(t')f_{2\sigma''}(t')\rangle\rangle. \quad (12)
\end{aligned}$$

In weak coupling approximation and with small level discrepancy ϵ, the correlation Green's functions in the isolated two dot system are:

$$
\begin{aligned}
G^{<0}_{eii\sigma}(\omega) &= 2\pi i \rho_{ii\sigma}\delta(\omega-\epsilon_d),\\
G^{>0}_{eii\sigma}(\omega) &= -2\pi i \rho_{00}\delta(\omega-\epsilon_d),\\
G^{<0}_{eij\sigma}(\omega) &= 2\pi i \rho_{ij\sigma}\delta(\omega-\epsilon_d), G^{>0}_{eij\sigma}(\omega)=0,\\
G^{<0}_{d11\sigma'\sigma\sigma''}(\omega) &= \delta_{\sigma\sigma''}2\pi i\rho_{dd\sigma\sigma'}\delta(\omega-\epsilon_d-U'),\\
G^{>0}_{d11\sigma'\sigma\sigma''}(\omega) &= -\delta_{\sigma\sigma''}2\pi i\rho_{11\sigma}\delta(\omega-\epsilon_d-U'),\\
G^{<0}_{d22\sigma'\sigma\sigma''}(\omega) &= \delta_{\sigma\sigma''}2\pi i\rho_{dd\sigma'\sigma}\delta(\omega-\epsilon_d-U'),\\
G^{>0}_{d22\sigma'\sigma\sigma''}(\omega) &= -\delta_{\sigma\sigma''}2\pi i\rho_{22\sigma}\delta(\omega-\epsilon_d-U'),\\
G'^{<0}_{d11\sigma'\sigma''\sigma}(\omega) &= \delta_{\sigma\sigma'}\delta_{\sigma\sigma''}2\pi i\rho_{dd\sigma\sigma}\delta(\omega-\epsilon_d-U'),\\
G'^{>0}_{d11\sigma'\sigma''\sigma}(\omega) &= -\delta_{\sigma\sigma'}\delta_{\sigma\sigma''}2\pi i\rho_{11\sigma}\delta(\omega-\epsilon_d-U'),\\
G'^{<0}_{d22\sigma\sigma'\sigma''}(\omega) &= \delta_{\sigma\sigma'}\delta_{\sigma\sigma''}2\pi i\rho_{dd\sigma\sigma}\delta(\omega-\epsilon_d-U'),\\
G'^{>0}_{d22\sigma\sigma'\sigma''}(\omega) &= -\delta_{\sigma\sigma'}\delta_{\sigma\sigma''}2\pi i\rho_{22\sigma}\delta(\omega-\epsilon_d-U'),\\
G^{<0}_{d21\sigma'\sigma''\sigma}(\omega) &= 0,\\
G^{>0}_{d21\sigma'\sigma''\sigma}(\omega) &= -\delta_{\sigma\sigma''}2\pi i\rho_{12\sigma}\delta(\omega-\epsilon_d-U'),\\
G'^{<0}_{d21\sigma\sigma'\sigma''}(\omega) &= 0,\\
G'^{>0}_{d21\sigma\sigma'\sigma''}(\omega) &= -\delta_{\sigma\sigma''}2\pi i\rho_{12\sigma}\delta(\omega-\epsilon_d-U'),\\
G''^{<0}_{d21\sigma'\sigma\sigma''}(\omega) &= 0,\\
G''^{>0}_{d21\sigma'\sigma\sigma''}(\omega) &= -\delta_{\sigma\sigma'}\delta_{\sigma\sigma''}2\pi i\rho_{12\sigma}\delta(\omega-\epsilon_d-U'),\\
G^{<0}_{d12\sigma\sigma'\sigma''}(\omega) &= 0,\\
G^{>0}_{d12\sigma\sigma'\sigma''}(\omega) &= -\delta_{\sigma\sigma'}\delta_{\sigma\sigma''}2\pi i\rho_{21\sigma}\delta(\omega-\epsilon_d-U'). \quad (13)
\end{aligned}
$$

Note that by using the lowest-order gradient expansion with slowly varying in the center-of-mass time T and rapidly varying in the relative time t and after the Fourier transformation from t to ω, the same results as the above can be acquired.

Inserting equation (13) into equations (5)–(10), we get the final quantum equations:

$$
\begin{aligned}
\dot\rho_{00} &= \sum_\sigma[-2\alpha_{1\sigma}\rho_{00}+\beta_{1\sigma}(\rho_{11\sigma}+\rho_{22\sigma})\\
&\quad+\beta_{2\sigma}\rho_{12\sigma}+\beta^*_{2\sigma}\rho_{21\sigma}],\\
\dot\rho_{11\sigma} &= \alpha_{1\sigma}\rho_{00}-(\beta_{1\sigma}+\sum_{\sigma'}\tilde\alpha_{1\sigma'})\rho_{11\sigma}-1/2(\beta_{2\sigma}+\tilde\alpha_{2\sigma})\\
&\quad\times\rho_{12\sigma}-1/2(\beta^*_{2\sigma}+\tilde\alpha^*_{2\sigma})\rho_{21\sigma}+\sum_{\sigma'}\tilde\beta_{1\sigma'}\rho_{dd\sigma\sigma'},\\
\dot\rho_{22\sigma} &= \alpha_{1\sigma}\rho_{00}-(\beta_{1\sigma}+\sum_{\sigma'}\tilde\alpha_{1\sigma'})\rho_{22\sigma}-1/2(\beta_{2\sigma}+\tilde\alpha_{2\sigma})\\
&\quad\times\rho_{12\sigma}-1/2(\beta^*_{2\sigma}+\tilde\alpha^*_{2\sigma})\rho_{21\sigma}+\sum_{\sigma'}\tilde\beta_{1\sigma'}\rho_{dd\sigma'\sigma},\\
\dot\rho_{21\sigma} &= \alpha_{2\sigma}\rho_{00}-1/2(\beta_{2\sigma}+\tilde\alpha_{2\sigma})(\rho_{11\sigma}+\rho_{22\sigma})\\
&\quad-(\beta_{1\sigma}+\sum_{\sigma'}\tilde\alpha_{1\sigma'})\rho_{21\sigma}+\tilde\beta_{2\sigma}\rho_{dd\sigma\sigma}\\
&\quad+i(\epsilon_1-\epsilon_2)\rho_{21\sigma},\\
\dot\rho_{dd\sigma\sigma} &= \tilde\alpha_{1\sigma}(\rho_{11\sigma}+\rho_{22\sigma})+\tilde\alpha_{2\sigma}\rho_{12\sigma}+\tilde\alpha^*_{2\sigma}\rho_{21\sigma}\\
&\quad-2\tilde\beta_{1\sigma}\rho_{dd\sigma\sigma},\\
\dot\rho_{dd\sigma\bar\sigma} &= \tilde\alpha_{1\sigma}\rho_{22\bar\sigma}+\tilde\alpha_{1\bar\sigma}\rho_{11\sigma}-(\tilde\beta_{1\sigma}+\tilde\beta_{1\bar\sigma})\rho_{dd\sigma\bar\sigma}. \quad (14)
\end{aligned}
$$

Here,

$$
\begin{aligned}
\alpha_{1\sigma} &= f_L(\epsilon_d)\Gamma_{L\sigma}+f_R(\epsilon_d)\Gamma_{R\sigma},\\
\beta_{1\sigma} &= [1-f_L(\epsilon_d)]\Gamma_{L\sigma}+[1-f_R(\epsilon_d)]\Gamma_{R\sigma},\\
\tilde\alpha_{1\sigma} &= f_L(\epsilon_d+U')\Gamma_{L\sigma}+f_R(\epsilon_d+U')\Gamma_{R\sigma},\\
\tilde\beta_{1\sigma} &= [1-f_L(\epsilon_d+U')]\Gamma_{L\sigma}+[1-f_R(\epsilon_d+U')]\Gamma_{R\sigma},\\
\alpha_{2\sigma} &= f_L(\epsilon_d)\Gamma_{L\sigma}e^{-i\varphi/2}+f_R(\epsilon_d)\Gamma_{R\sigma}e^{i\varphi/2},\\
\beta_{2\sigma} &= [1-f_L(\epsilon_d)]\Gamma_{L\sigma}e^{-i\varphi/2}+[1-f_R(\epsilon_d)]\Gamma_{R\sigma}e^{i\varphi/2},\\
\tilde\alpha_{2\sigma} &= f_L(\epsilon_d+U')\Gamma_{L\sigma}e^{-i\varphi/2}+f_R(\epsilon_d+U')\Gamma_{R\sigma}e^{i\varphi/2},\\
\tilde\beta_{2\sigma} &= [1-f_L(\epsilon_d+U')]\Gamma_{L\sigma}e^{-i\varphi/2}\\
&\quad+[1-f_R(\epsilon_d+U')]\Gamma_{R\sigma}e^{i\varphi/2}. \quad (15)
\end{aligned}
$$

Complemented with the completeness relation $\rho_{00}+2\rho_{11}+2\rho_{22}+2\rho_{dd}+2\rho_{d\bar d}=1$, the closed equations equation (14) can be solved to determine the expectation values in steady states.

3 Discussion

In the equilibrium state, only the diagonal distribution probabilities are nonzero, $\rho_{00}=1/Z, \rho_{11\sigma}=e^{-\beta\epsilon_d}/Z, \rho_{22\sigma}=e^{-\beta\epsilon_d}/Z, \rho_{dd\sigma\sigma'}=e^{-\beta(2\epsilon_d+U')}/Z$ and $Z=1+4e^{-\beta\epsilon_d}+4e^{-\beta(2\epsilon_d+U')}$, which can be readily gotten from the above equations set with $f_L(\epsilon_d)=f_R(\epsilon_d)=(e^{\beta\epsilon_d}+1)^{-1}, f_L(\epsilon_d+U')=f_R(\epsilon_d+U')=\{e^{\beta(\epsilon_d+U')}+1\}^{-1}$ supposing the equilibrium chemical potential $\mu_L=\mu_R=0$. The results meet the classical Boltzmann distribution in weak coupling limit. It is obvious that the occupation number of dot 1 with certain spin $c^\dagger_{1\sigma}c_{1\sigma}=\rho_{11\sigma}+\sum_{\sigma'}\rho_{dd\sigma\sigma'}$ has nothing to do with the magnetic flux Φ and the effect of dot level disparity ϵ is not taken into account, which is the same as dot 2, and in the spin symmetry space $\rho_{dd\sigma\sigma}=\rho_{dd\sigma\bar\sigma}$.

Applying external voltage on the two leads, the whole system is driven out of equilibrium and off-diagonal elements play a vital role in transport, whereupon some novel features arise. In the spin symmetry space $\alpha(\tilde\alpha)_{1(2)\sigma}$, $\beta(\tilde\beta)_{1(2)\sigma}$, $\Gamma_{L(R)\sigma}$ are all independent of spin, so for simplicity we subtract their spin index σ. After straightforward derivation, the solutions for these closed equations and the final formula of current in steady states are obtained in Appendix. We define $\eta\equiv(I_{\max}-I_{\min})/(I_{\max}+I_{\min})$ for the description of the interference strength. If $\eta=1$, the transport is totally coherent, if $\eta=0$, the transport is totally incoherent, and if $1>\eta>0$, the transport is partially coherent. In general cases, partially coherent transport takes place. We see that the occupation number of the dot 1(2) shows explicit AB oscillations originating from the interference of two dot states $\rho_{12\sigma}$ and $\rho_{21\sigma}$, because phase related coefficients $\alpha_{2\sigma}$, $\beta_{2\sigma}$, $\tilde\alpha_{2\sigma}$, $\tilde\beta_{2\sigma}$ in the equation (14) are only associated with the existence of $\rho_{12\sigma}$ and $\rho_{21\sigma}$, which also induce the difference between $\rho_{dd\sigma\sigma}$ and $\rho_{dd\sigma\bar\sigma}$ in the nonequilibrium state.

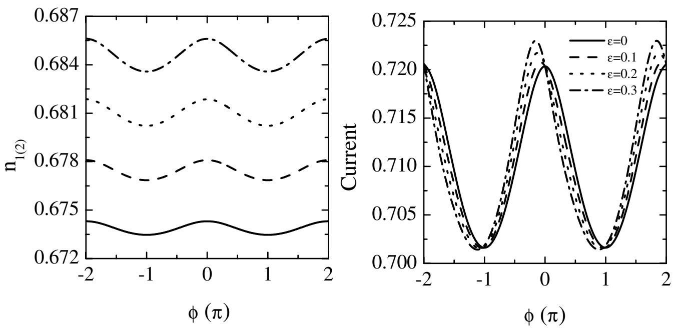

Fig. 2. Left: AB oscillations of dot population $n_{1(2)}$ with interdot correlation $U' = 0$ and energy disparity $\epsilon = 0$ at temperature $k_BT = 20$ and bias voltage $eV = 50$ for different average level ϵ_d: -1(solid), -1.5(dash), -2(dot), -2.5(dash dot). Right: AB oscillations of current for different energy disparity ϵ with interdot correlation $U' = 20$ and average level $\epsilon_d = -1$ at temperature $k_BT = 20$ and bias voltage $eV = 50$. $e = \hbar = \Gamma = 1$.

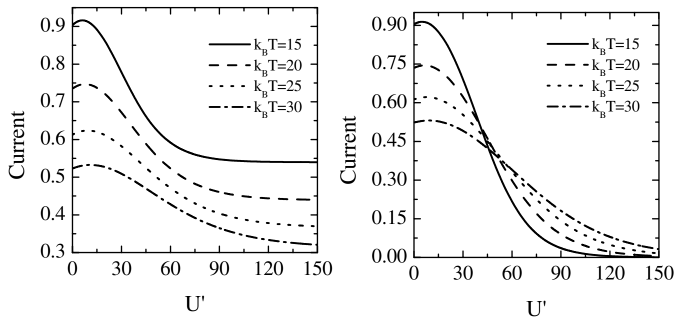

Fig. 3. Current versus interdot correlation U' with various temperatures for average dot level $\epsilon_d = -1$, energy disparity $\epsilon = 0$, and bias voltage $eV = 50$ at $\varphi = 0$ (left) and at $\varphi = \pi$(right). $e = \hbar = \Gamma = 1$.

Note that the temperature is assumed higher than the Kondo temperature so that the Kondo correlation which is attributed to the strong coupling between leads and dots can be negligible. In the sequential tunneling picture, the tunneling contribution to transport is up to first order in Γ in the Coulomb blockade regime. We can directly calculate the current from equation (21) which is valid for arbitrary bias voltage eV, interdot repulsion U', and dot energy position ϵ_d in broad ranges and for tiny level variation ϵ between two dots. For convenience, the symmetric bias voltage $\mu_L = -\mu_R = eV/2$ is applied, then all the results are invariant under the reversals of magnetic flux $\varphi \to -\varphi$ and of bias voltage $eV \to -eV$ simultaneously. This is in accordance with the Onsager relation for two-terminal setups [12]. In general, the occupation number of dot 1(2) and the current through the system all have AB oscillations, combinations of $\cos\varphi$ and $\sin\varphi$, with a period of Φ_0, and the amplitudes depend on the resonant energy level ϵ_d and $\epsilon_d + U'$ relative to the chemical potentials of two leads and small disparity amplitude ϵ. It is also supposed that $k_BT \gg \Gamma$, $|\epsilon_1|$, $|\epsilon_2|$, and $\Gamma \gg |\epsilon|$, so the lowest-order transport dominates. In Figure 2 we plot AB oscillations of the population in dot with different level positions and the current with different energy discrepancys respectively. For two identical dots $\epsilon = 0$, all oscillations are symmetric about $\varphi = 0$ including only $\cos\varphi$ with larger amplitudes for deeper levels. The population reaches maximum at $\varphi = 0$ and minimum at $\varphi = \pi$. For two distinct dots $\epsilon \neq 0$, the symmetry about $\varphi = 0$ is lost, which gets more apparent with larger ϵ for oscillations following $\sin\varphi$ take effect, and the extremes at $\varphi = 0$ and $\varphi = \pi$ also disappear. Figure 3 shows current versus interdot correlation with various temperatures for $\varphi = 0$ and $\varphi = \pi$. We can see that at finite temperature and finite bias the interdot correlation does not always inhibit transport. Only when U' is large enough, the prohibition effect appears, especially at $\varphi = \pi$, and this is mitigated by raising the temperature.

In the following two specially simple cases with equal energy levels of two dots are under consideration. First is that doubly occupied states are forbidden by infinite interdot Coulomb repulsion U', such that $\rho_{dd\sigma\sigma'} = 0$, and $\tilde{\alpha}_1 = 0, \tilde{\alpha}_2 = 0$. The steady solutions are

$$\begin{aligned}
\rho_{11} &= \frac{(2 - f_L - f_R)[(1 - f_L)f_R + (1 - f_R)f_L]}{2(f_L + f_R - 3f_Lf_R + 1)(2 - f_L - f_R)}, \\
\rho_{12} &= \frac{f_L - f_R}{2(f_L + f_R - 3f_Lf_R + 1)}\left[\frac{f_R - f_L}{2 - f_L - f_R}\cos\frac{\varphi}{2}\right. \\
&\quad \left. + i\sin\frac{\varphi}{2}\right], \\
\rho_{00} &= \frac{f_Lf_R - f_L - f_R + 1}{f_L + f_R - 3f_Lf_R + 1},
\end{aligned} \tag{16}$$

and the current is

$$I = \frac{2\pi e\Gamma}{h}\frac{2(f_L - f_R)(1 - f_R)(1 - f_L)}{(f_L + f_R - 3f_Lf_R + 1)(2 - f_L - f_R)}(1 + \cos\varphi). \tag{17}$$

It is evident that in this case at arbitrary voltage and dot level, with $\varphi = 0, \pm 2\pi, \pm 4\pi, ...$, the current reaches the maximum, and phase locking always happens, while with $\varphi = \pm\pi, \pm 3\pi, \pm 5\pi, ...$, the current vanishes and the interference between two dots is completely destructive. It is worthy to point out that reference [12] has gotten the same result, but they discussed spinless electrons and finite interdot Coulomb repulsion, while in our method including the spin degeneracy, a finite interdot Coulomb repulsion will induce a finite current at flux equal to odd multiples of π. Considering only the first order of Γ in sequential tunneling regime, we get the total coherence.

Second is that empty state is forbidden in deep level status with ϵ_d far below the Fermi level and $\epsilon_d + U'$ just above it in equilibrium state, so that $\rho_{00} = 0$, and

$$\rho_{11} = \frac{(2-\tilde{f}_L-\tilde{f}_R)[(1-\tilde{f}_L)(2\tilde{f}_R+\tilde{f}_L-\tilde{f}_R\cos\varphi)+(1-\tilde{f}_R)(2\tilde{f}_L+\tilde{f}_R-\tilde{f}_L\cos\varphi)]}{4(\tilde{f}_L+\tilde{f}_R)(3-\cos\varphi)(2-\tilde{f}_L-\tilde{f}_R)+2(\tilde{f}_L-\tilde{f}_R)^2(1-\cos\varphi)},$$

$$\rho_{12} = \frac{1}{4(\tilde{f}_L+\tilde{f}_R)}\Bigg[(2-\tilde{f}_L-\tilde{f}_R)\cos\frac{\varphi}{2}+i\sin\frac{\varphi}{2}(\tilde{f}_L-\tilde{f}_R)-\left(4\cos\frac{\varphi}{2}-2i\sin\frac{\varphi}{2}\frac{\tilde{f}_R-\tilde{f}_L}{2-\tilde{f}_L-\tilde{f}_R}\right)$$
$$\frac{(2-\tilde{f}_L-\tilde{f}_R)[(1-\tilde{f}_L)(2\tilde{f}_R+\tilde{f}_L-\tilde{f}_R\cos\varphi)+(1-\tilde{f}_R)(2\tilde{f}_L+\tilde{f}_R-\tilde{f}_L\cos\varphi)]}{4(\tilde{f}_L+\tilde{f}_R)(3-\cos\varphi)(2-\tilde{f}_L-\tilde{f}_R)+2(\tilde{f}_L-\tilde{f}_R)^2(1-\cos\varphi)}\Bigg],$$

$$\rho_{dd} = \frac{1}{2}-\frac{(4-\tilde{f}_L-\tilde{f}_R)[(1-\tilde{f}_L)(2\tilde{f}_R+\tilde{f}_L-\tilde{f}_R\cos\varphi)+(1-\tilde{f}_R)(2\tilde{f}_L+\tilde{f}_R-\tilde{f}_L\cos\varphi)]}{4(\tilde{f}_L+\tilde{f}_R)(3-\cos\varphi)(2-\tilde{f}_L-\tilde{f}_R)+2(\tilde{f}_L-\tilde{f}_R)^2(1-\cos\varphi)},$$

$$\rho_{d\bar{d}} = \frac{(\tilde{f}_L+\tilde{f}_R)[(1-\tilde{f}_L)(2\tilde{f}_R+\tilde{f}_L-\tilde{f}_R\cos\varphi)+(1-\tilde{f}_R)(2\tilde{f}_L+\tilde{f}_R-\tilde{f}_L\cos\varphi)]}{4(\tilde{f}_L+\tilde{f}_R)(3-\cos\varphi)(2-\tilde{f}_L-\tilde{f}_R)+2(\tilde{f}_L-\tilde{f}_R)^2(1-\cos\varphi)}, \quad (18)$$

$$I = \frac{\pi e\Gamma}{h}\frac{\tilde{f}_L-\tilde{f}_R}{\tilde{f}_L+\tilde{f}_R}\Bigg\{1+2(\tilde{f}_L+\tilde{f}_R)+(1-\tilde{f}_L-\tilde{f}_R)\cos\varphi-[(4-\tilde{f}_L-\tilde{f}_R)$$
$$+(4-3\tilde{f}_L-3\tilde{f}_R)\cos\varphi]\frac{(1-\tilde{f}_L)(2\tilde{f}_R+\tilde{f}_L-\tilde{f}_R\cos\varphi)+(1-\tilde{f}_R)(2\tilde{f}_L+\tilde{f}_R-\tilde{f}_L\cos\varphi)}{2(\tilde{f}_L+\tilde{f}_R)(3-\cos\varphi)(2-\tilde{f}_L-\tilde{f}_R)+(\tilde{f}_L-\tilde{f}_R)^2(1-\cos\varphi)}\Bigg\}, \quad (19)$$

$\beta_1 = 0$, $\beta_2 = 0$. The steady solutions are

see equation (18) above

and the current is

see equation (19) above

which is partially coherent, and the AB oscillations for the population in dots and the current all follow the $\cos\varphi$ form.

4 Summary

To conclude, we have studied the quantum transport through a parallel double-dot structure with the Aharonov-Bohm magnetic flux infiltering the two path closed region. Infinite intradot and arbitrary interdot Coulomb repulsion have been considered by employing the slave-boson technique introduced by Zou and Anderson. In weak coupling and sequential tunneling regime we use the "classical" quantum rate equations combined with nonequilibrium Green's functions to determine the expectation values of projection operators for the density matrix and to calculate the current flowing through the system of stationary state. The results show that external bias voltage induced superposition of two dots states is the cause of phase coherence of AB oscillations of population of each dot and the current as a function of magnetic flux having a period of Φ_0. Two simplest cases are discussed as examples for two identical dots. We find that if there are no doubly occupied states, the current is totally coherent – completely destructive interference exists with φ odd multiples of π and maximum always reaches with φ even multiples of π; if there is no empty state, the population and the current are partially coherent and amplitudes depend on the resonant energy levels relative to the chemical potentials of two reservoirs.

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5 Appendix

The solutions of the expectation values of the projection operators for the density matrix in stationary states are: $\rho_{11\sigma} = \rho_{11\bar{\sigma}} = \rho_{11}$, $\rho_{22\sigma} = \rho_{22\bar{\sigma}} = \rho_{22}$, $\rho_{21\sigma} = \rho_{21\bar{\sigma}} = \rho_{21}$, $\rho_{12\sigma} = \rho_{12\bar{\sigma}} = \rho_{12}$, $\rho_{dd\sigma\sigma} = \rho_{dd\bar{\sigma}\bar{\sigma}} = \rho_{dd}$, $\rho_{dd\sigma\bar{\sigma}} = \rho_{dd\bar{\sigma}\sigma} = \rho_{d\bar{d}}$,

$$\begin{aligned}
\rho_{00} &= (C_2A_1 - C_1A_2)/(B_1A_2 - B_2A_1),\\
\rho_{11} &= (C_2B_1 - C_1B_2)/[2(A_1B_2 - A_2B_1)],\\
\rho_{21} &= \{(2\alpha_2 - \tilde{\beta}_2)\rho_{00} - [\tilde{\alpha}_2 + \beta_2\\
&\quad +(2+\tilde{\alpha}_1/\tilde{\beta}_1)\tilde{\beta}_2](\rho_{11}+\rho_{22})\\
&\quad +\tilde{\beta}_2\}/[2(\beta_1 + 2\tilde{\alpha}_1 + i\epsilon)],\\
\rho_{dd} &= [1 - \rho_{00} - (2+\tilde{\alpha}_1/\tilde{\beta}_1)(\rho_{11}+\rho_{22})]/2,\\
\rho_{d\bar{d}} &= \tilde{\alpha}_1(\rho_{11}+\rho_{22})/(2\tilde{\beta}_1),
\end{aligned} \quad (20)$$

and $\rho_{22} = \rho_{11}$, $\rho_{12} = \rho_{21}^*$. Here,

$$A_1 = 4(\Delta^2 + \epsilon^2) - \Delta(\tilde{f}_L + \tilde{f}_R \cos\varphi)[1 + \tilde{f}_L - f_L + \gamma(1 - \tilde{f}_L)] - \Delta(\tilde{f}_R + \tilde{f}_L \cos\varphi)[1 + \tilde{f}_R - f_R + \gamma(1 - \tilde{f}_R)] + \epsilon \sin\varphi[(\tilde{f}_R - \tilde{f}_L)(1 + \gamma) + \tilde{f}_L f_R - \tilde{f}_R f_L],$$

$$B_1 = \Delta[2\tilde{f}_L(f_L + f_R \cos\varphi) + 2\tilde{f}_R(f_R + f_L \cos\varphi) - (1 - \tilde{f}_L)(\tilde{f}_L + \tilde{f}_R \cos\varphi) - (1 - \tilde{f}_R)(\tilde{f}_R + \tilde{f}_L \cos\varphi)] + (2 - \tilde{f}_L - \tilde{f}_R)(\Delta^2 + \epsilon^2) - \epsilon \sin\varphi[2(\tilde{f}_R f_L - \tilde{f}_L f_R) + \tilde{f}_L - \tilde{f}_R],$$

$$C_1 = -(2 - \tilde{f}_L - \tilde{f}_R)(\Delta^2 + \epsilon^2) + \Delta[(1 - \tilde{f}_L)(\tilde{f}_L + \tilde{f}_R \cos\varphi) + (1 - \tilde{f}_R)(\tilde{f}_R + \tilde{f}_L \cos\varphi)] + \epsilon \sin\varphi(\tilde{f}_L - \tilde{f}_R),$$

$$A_2 = (2 - f_L - f_R)(\Delta^2 + \epsilon^2) - \Delta(1 - f_L)[1 + \tilde{f}_L - f_L + \cos\varphi(1 + \tilde{f}_R - f_R) + \gamma(1 - \tilde{f}_L + \cos\varphi - \cos\varphi \tilde{f}_R)] - \Delta(1 - f_R)[1 + \tilde{f}_R - f_R + \cos\varphi(1 + \tilde{f}_L - f_L) + \gamma(1 - \tilde{f}_R + \cos\varphi - \cos\varphi \tilde{f}_L)] + \epsilon \sin\varphi[(1 - f_R)(\tilde{f}_L + \gamma - \gamma \tilde{f}_L) - (1 - f_L)(\tilde{f}_R + \gamma - \gamma \tilde{f}_R)],$$

$$B_2 = \Delta(2f_L + \tilde{f}_L - 1)(1 - f_L + \cos\varphi - \cos\varphi f_R) + \Delta(2f_R + \tilde{f}_R - 1)(1 - f_R + \cos\varphi - \cos\varphi f_L) - 2(f_L + f_R)(\Delta^2 + \epsilon^2) - \epsilon \sin\varphi[(1 - f_R)(1 + \tilde{f}_L) - (1 - f_L)(1 + \tilde{f}_R)],$$

$$C_2 = \Delta[(1 - f_L)(1 - \tilde{f}_L + \cos\varphi - \cos\varphi \tilde{f}_R) + (1 - f_R)(1 - \tilde{f}_R + \cos\varphi - \cos\varphi \tilde{f}_L)] - \epsilon \sin\varphi[(1 - f_R)(1 - \tilde{f}_L) - (1 - f_L)(1 - \tilde{f}_R)],$$

and the current formula becomes

$$I = \frac{2\pi e\Gamma}{h}[(f_L - f_R)(2\rho_{00} + \rho_{11} + \rho_{22}) + \cos\varphi/2(f_L + \tilde{f}_L - f_R - \tilde{f}_R)(\rho_{12} + \rho_{21}) - i \sin\varphi/2(f_L + \tilde{f}_L + f_R + \tilde{f}_R - 2)(\rho_{12} - \rho_{21}) + 2(\tilde{f}_L - \tilde{f}_R)(\rho_{11} + \rho_{22} + \rho_{dd} + \rho_{d\bar{d}})], \tag{21}$$

where $\Delta \equiv 2 - f_L - f_R + 2\tilde{f}_L + 2\tilde{f}_R$, $\gamma \equiv (4 - \tilde{f}_L - \tilde{f}_R)/(2 - \tilde{f}_L - \tilde{f}_R)$, and $f_{L(R)} \equiv f_{L(R)}(\epsilon_d) = \{e^{\beta(\epsilon_d - \mu_{L(R)})} + 1\}^{-1}$, $\tilde{f}_{L(R)} \equiv f_{L(R)}(\epsilon_d + U') = \{e^{\beta(\epsilon_d + U' - \mu_{L(R)})} + 1\}^{-1}$.

References

1. A. Yacoby, M. Heiblum, D. Mahalu, H. Shtrikman, Phys. Rev. Lett. **74**, 4047 (1995)
2. R. Schuster, E. Buks, M. Heiblum, D. Mahalu, V. Umansky, H. Shtrikman, Nature (London) **385**, 417 (1997)
3. Y. Ji, M. Heiblum, D. Sprinzak, D. Mahalu, H. Shtrikman, Science **290**, 779 (2000); Y. Ji, M. Heiblum, H. Shtrikman, Phys. Rev. Lett. **88**, 076601 (2002)
4. W.G. van de Wiel, S. De Franceschi, T. Fujisawa, J.M. Elzerman, S. Tarucha, L.P. Kouwenhoven, Science **289**, 2105 (2000)
5. K. Kobayashi, H. Aikawa, S. Katsumoto, Y. Iye, Phys. Rev. Lett. **88**, 256806 (2002)
6. A.W. Holleitner, C.R. Decker, H. Qin, K. Eberl, R.H. Blick, Phys. Rev. Lett. **87**, 256802 (2001)
7. A.W. Holleitner, R.H. Blick, A.K. Hüttel, K. Eberl, J.P. Kotthaus, Science **297**, 70 (2002)
8. R.H. Blick, A.K. Hüttel, A.W. Holleitner, E.M. Höhberger, H. Qin, J. Kirschbaum, J. Weber, W. Wegscheider, M. Bichler, K. Eberl, J.P. Hotthaus, Physica E **16**, 76 (2003)
9. T.V. Shahbazyan, M.E. Raikh, Phys. Rev. B **49**, 17123 (1994)
10. B. Kubala, J. König, Phys. Rev. B **65**, 245301 (2002); B. Kubala, J. König, Phys. Rev. B **67**, 205303 (2003)
11. K. Kang, S.Y. Cho, preprint: `cond-mat/0210009`
12. J. König, Y. Gefen, Phys. Rev. Lett. **86**, 3855 (2001); J. König, Y. Gefen, Phys. Rev. B **65**, 045316 (2002)
13. Z.T. Jiang, J.Q. You, S.B. Bian, H.Z. Zheng, Phys. Rev. B **66**, 205306 (2002)
14. M.L. Ladrón de Guevara, F. Claro, Pedro A. Orellana, Phys. Rev. B **67**, 195335 (2003)
15. Z.M. Bai, M.F. Yang, Y.C. Chen, preprint: `cond-mat/0307071`
16. H. Akera, Phys. Rev. B **47**, 6835 (1993)
17. D. Loss, E.V. Sukhorukov, Phys. Rev. Lett. **84**, 1035 (2000)
18. D. Boese, W. Hofstetter, H. Schoeller, Phys. Rev. B **64**, 125309 (2001)
19. L.G. Mourokh, N.J.M. Horing, Phys. Rev. B **66**, 085332 (2002)
20. D. Boese, W. Hofstetter, H. Schoeller, Phys. Rev. B **66**, 125315 (2002)
21. Yu. V. Nazarov, Physica B **189**, 57 (1993).
22. A.N. Korotkov, Phys. Rev. B **60**, 5737 (1999); A.N. Korotkov, Phys. Rev. B **63**, 115403 (2001); A.N. Korotkov, Phys. Rev. B **67**, 75303 (2003)
23. H.S. Goan, G.J. Milburn, H.M. Wisernan, H.B. Sun, Phys. Rev. B **63**, 125326 (2001)
24. B. Elattari, S.A. Gurvitz, Phys. Rev. Lett. **84**, 2047 (2000); B. Elattari, S.A. Gurvitz, Phys. Rev. A **62**, 32102 (2000)
25. T.H. Stoof, Yu. V. Nazarov, Phys. Rev. B **55**, 1050 (1999); B. L. Hazelzet, M.R. Wegewijs, T.H. Stoof, Yu. V. Nazarov, Phys. Rev. B **63**, 165313 (2001)
26. B. Dong, H.L. Cui, X.L. Lei, preprint: `cond-mat/0308387`, Phys. Rev. B (in press)
27. Z. Zou, P.W. Anderson, Phys. Rev. B **37**, 627 (1988)
28. J.H. Davies, S. Hershfield, P. Hyldgaard, J.W. Wilkins, Phys. Rev. B **47**, 4603 (1993)
29. J.C. Le Guillou, E. Ragoucy, Phys. Rev. B **52**, 2403 (1995)